Expand and simplify $(t^2 - 2\sqrt{t})^4$.

SCORE: _____ / 6 PTS

You may write the coefficients in your final answer in factored form, as shown in lecture.

The coefficients in your final answer must **NOT** contain division, ! nor $C(n, r)$ (or equivalent) notation.

NOTE: Do NOT use your calculator's ! nor $C(n, r)$ features.

$$(t^2)^4 + \overset{1}{\underline{4(t^2)^3(-2\sqrt{t})}} + \overset{1}{\underline{6(t^2)^2(-2\sqrt{t})^2}} + \overset{1}{\underline{4(t^2)(-2\sqrt{t})^3}} + (-2\sqrt{t})^4$$

$$= \underbrace{t^8}_{\frac{1}{2}} \underbrace{- 8t^{\frac{13}{2}}}_{\frac{1}{2}} \underbrace{+ 24t^5}_{\frac{1}{2}} \underbrace{- 32t^{\frac{7}{2}}}_{\frac{1}{2}} \underbrace{+ 16t^2}_{1}$$

Consider the expansion of $(3x^{10}+\frac{5}{x^2})^{24}$. SCORE: _____ / 9 PTS

For the questions below, you may write the coefficients in your final answers in factored form, as shown in lecture.
The coefficients in your final answers must **NOT** contain division, ! nor $C(n,r)$ (or equivalent) notation.

NOTE: Do NOT use your calculator's ! nor $C(n,r)$ features.

$\frac{1}{2}$ POINT EACH EXCEPT AS NOTED

[a] Find the fifth term in the expansion.

$$\binom{24}{4}(3x^{10})^{24-4}(\tfrac{5}{x^2})^4 = \boxed{\frac{24!}{4!20!}}3^{20}x^{200}5^4x^{-8} = \boxed{\frac{24\cdot23\cdot22\cdot21\cdot20!}{4\cdot3\cdot2\cdot1\cdot20!}}3^{20}5^4x^{192} = 23\cdot22\cdot21\cdot3^{20}5^4x^{192}$$

[b] Find the coefficient of x^{12} in the expansion.

2

$$\sum_{r=0}^{24}\binom{24}{r}(3x^{10})^{24-r}(\tfrac{5}{x^2})^r = \sum_{r=0}^{24}\binom{24}{r}3^{24-r}x^{10(24-r)}5^rx^{-2r} = \sum_{r=0}^{24}\binom{24}{r}3^{24-r}5^rx^{240-12r}$$

$$x^{240-12r}=x^{12} \Rightarrow 240-12r=12 \Rightarrow 20-r=1 \Rightarrow r=19$$

$$\binom{24}{19}3^{24-19}5^{19} = \binom{24}{19}3^5 5^{19} = \boxed{\frac{24!}{19!5!}}3^5 5^{19} = \boxed{\frac{24\cdot23\cdot22\cdot21\cdot20\cdot19!}{5\cdot4\cdot3\cdot2\cdot1\cdot19!}}3^5 5^{19} = 23\cdot22\cdot21\cdot4\cdot3^5 5^{19}$$

[c] Find the coefficient of x^{30} in the expansion.

$$x^{240-12r}=x^{30} \Rightarrow 240-12r=30 \Rightarrow 40-2r=5 \Rightarrow r=\tfrac{35}{2}\text{ which is not an integer}$$

No x^{30} term, so coefficient $=0$

Prove that $\sum_{i=1}^{n}[(i-1)\cdot(i-1)!+1]=n!+n-1$ for all positive integers n **<u>using mathematical induction</u>**. **SCORE: _____ / 15 PTS**

Basis case: $\sum_{i=1}^{1}[(i-1)\cdot(i-1)!+1] \;=\; 0\cdot 0!+1 \;=\; 1 \;=\; 1!+1-1$

GRADED BY ME

Inductive step: Assume that $\sum_{i=1}^{k}[(i-1)\cdot(i-1)!+1]=k!+k-1$ for some arbitrary integer $k\geq 1$

$$\boxed{\text{Prove that } \sum_{i=1}^{k+1}[(i-1)\cdot(i-1)!+1]=(k+1)!+k+1-1=(k+1)!+k}$$

$$\sum_{i=1}^{k+1}[(i-1)\cdot(i-1)!+1]$$

$$=\sum_{i=1}^{k}[(i-1)\cdot(i-1)!+1] \quad + \quad k\cdot k!+1$$

$$=k!+k-1 \quad + \quad k\cdot k!+1$$

$$=k!+k\cdot k!+k$$

$$=(1+k)\cdot k!+k$$

$$=(k+1)!+k$$

So, by mathematical induction, $\sum_{i=1}^{n}[(i-1)\cdot(i-1)!+1]=n!+n-1$ for all positive integers n